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Second moment of quark structure function of the ρ-meson in QCD sum rules

Armen Oganesian and Alexander Samsonov

Institute of Theoretical and Experimental Physics
State Research Center, Moscow, Russia
E-mail: armen@heron.itep.ru, sams@heron.itep.ru

Abstract: The second moments of the quark structure functions of the longitudinally $\mathcal{M}_\rho^L$ and transversally $\mathcal{M}_\rho^T$ polarized ρ-mesons are calculated in the framework of QCD sum rules in external fields. The operators of dimension 4 and 6 are taken into account. The results are ($\mu^2 = 1\,\mathrm{GeV}^2$): $\mathcal{M}_\rho^L = 0.84 \pm 0.08$, $\mathcal{M}_\rho^T = 0.5 \pm 0.1$. The large difference between the values of $\mathcal{M}_\rho^L$ and $\mathcal{M}_\rho^T$ indicates that the ρ-meson polarization strongly influences the internal structure. In particular, in the case of the longitudinally polarized ρ-meson the gluon sea is found to be strongly suppressed, it is less than $20 \div 25\%$, whereas in usual hadron case — about $40 \div 50\%$.

Keywords: Sum Rules, QCD.

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1. Introduction

Quark structure functions determine quark distributions in the hadrons and thus describe their dynamical properties. The structure functions of some hadrons (pion or nucleon) can be obtained from experimental data, while for other hadrons the necessary experimental information is absent. That is why theoretical investigation of the structure functions is of great interest.

The method of calculation of the structure functions in the framework of QCD sum rules [1] was suggested in [2]. In [3, 4] this method was generalized and the structure functions of the pion and ρ-meson were found.

However, in the framework of the suggested technique the structure functions can not be calculated at the values of the Bjorken variable x, close to one or zero [2]. Some additional assumptions about structure functions behaviour in the region of large ($0.7 \lesssim x < 1$) and small ($0 < x \lesssim 0.2$) values of the Bjorken variable were used to describe the structure functions in the whole interval of x [4].

From this point of view the direct non-model calculation of second moments of the structure functions in QCD sum rules is very desirable. The comparison of the values obtained with those found in [4] allows one to improve the structure functions behaviour at small and large x. Also it is very interesting to verify the statement, proposed in [4], about large difference between the structure functions of the longitudinally and transversally polarized ρ-meson.

Furthermore, the values of the second moment are of interest themselves, since they can be compared with those obtained from lattice simulations.

In the present paper we calculate the second moment of the quark structure function of the longitudinally and transversally polarized ρ-meson, using QCD sum rules in the external constant field. This technique was developed in [5]. In this way the second moments of the structure functions of the nucleon, pion, and kaon ([6], [7] and [8], where the quark masses were taken into account) were calculated.

In our paper u- and d-quarks are considered as massless, α_s-corrections are neglected.

2. Phenomenological part of the sum rule

Let us consider the quark part of the symmetrized energy-momentum tensor:

$$T_{\mu\nu} = \frac{i}{2} \sum_{\psi=u,d} \overline{\psi} \left(\overrightarrow{D}_{\langle\{\mu}\gamma_{\nu\}} - \overleftarrow{D}_{\{\mu}\gamma_{\nu\}\rangle} \right) \psi \,, \tag{2.1}$$

where the curly brackets mean symmetrization over the indices, the angle brackets remind that the trace must be removed, $\overrightarrow{D_\mu} = \overrightarrow{\partial_\mu} - igA^n_\mu t^n$, $\overleftarrow{D_\mu} = \overleftarrow{\partial_\mu} + igA^n_\mu t^n$, t^n are Gell-Mann matrices. Hereafter we denote $\overrightarrow{D}_\mu$ simply as D_μ.

The second moment $\mathcal{M}_\rho$ of the ρ-meson structure function is known to be related to $T_{\mu\nu}$ in the following way (see, for example, [8]):

$$\langle\rho|T_{\mu\nu}|\rho\rangle = 2\mathcal{M}_\rho p_\mu p_\nu \,, \tag{2.2}$$

where p_μ is the ρ-meson momentum, the traces are removed.

In order to calculate $\mathcal{M}_\rho$ in the QCD sum rules technique one should introduce a constant external field with a structure corresponded to the operator $T_{\mu\nu}$ (2.1). That is why we consider the symmetric tensor field with zero trace $S_{\mu\nu}$, $S_{\mu\nu} = S_{\nu\mu}$, $S_{\mu\mu} = 0$. Interaction with this field is determined by the additional term in the lagrangian:

$$\Delta\mathcal{L} = -S_{\mu\nu}T_{\mu\nu} \,. \tag{2.3}$$

In this paper we consider the correlator of two vector currents with the ρ-meson quantum numbers in the external field $S_{\mu\nu}$:

$$\Pi_{\mu\nu}(p) = i \int d^4x \, e^{ipx} \langle T(j_\mu(x) j^+_\nu(0)) \rangle_S \,, \tag{2.4}$$

where subscript S means presence of the external field.

In this correlator the vector current has the form: $j_\mu = \overline{u}\gamma_\mu d$, and its matrix element is

$$\langle\rho^+|j_\mu|0\rangle = (m^2_\rho/g_\rho)e_\mu \,, \tag{2.5}$$

where m_ρ is the ρ-meson mass, g_ρ is the ρ-γ coupling constant, $g^2_\rho/(4\pi) = 1.27$, and e_μ is the ρ-meson polarization vector.

In the limit of weak external field we consider only linear in $S_{\mu\nu}$ terms in the correlator $\Pi_{\mu\nu}$ (2.4):

$$\Pi_{\mu\nu} = \Pi^0_{\mu\nu} + \Pi_{\mu\nu\rho\lambda} S_{\rho\lambda} \,. \tag{2.6}$$

Let us construct sum rules for invariant functions at certain kinematical structures of $\Pi_{\mu\nu\rho\lambda}$ (2.6). According to the QCD sum rules technique, we represent these functions in two ways. On the one hand, we calculate them at $p^2 < 0$ as the operator product expansion series. On the other hand, we saturate their dispersion relations by the contributions of physical states. Here we use the simplest model of physical spectrum, which contains the lowest resonance and continuum. After equating of these representations the required sum rules appear.

Therefore, first of all one should choose the kinematical structures. Let us suppose for a while that $\Pi_{\mu\nu\rho\lambda}$ is saturated by the ρ-meson only. We denote such $\Pi_{\mu\nu\rho\lambda}$ as $\Pi^\rho_{\mu\nu\rho\lambda}$. One can extract an arbitrary invariant function Π^i from $\Pi^\rho_{\mu\nu\rho\lambda} S_{\rho\lambda}$, multiplying it by some projection operator $A^i_{\mu\nu}$: $\Pi^i = \Pi^\rho_{\mu\nu\rho\lambda} S_{\rho\lambda} A^i_{\mu\nu}$. The dispersion relation for Π^i has the form (we take into account (2.5)):

$$\begin{aligned}
\Pi^i = \Pi^\rho_{\mu\nu\rho\lambda} S_{\rho\lambda} A^i_{\mu\nu} &= \sum_{r,r'} \frac{\langle 0|j_\mu|\rho^r\rangle \langle \rho^r|T_{\rho\lambda}|\rho^{r'}\rangle \langle \rho^{r'}|j_\nu|0\rangle}{(m_\rho^2 - p^2)^2} S_{\rho\lambda} A^i_{\mu\nu} \\
&= \frac{m_\rho^4}{g_\rho^2} \sum_{r,r'} \frac{e^r_\mu e^r_\tau W_{\tau\sigma\lambda\rho} e^{r'}_\sigma e^{r'}_\nu}{(m_\rho^2 - p^2)^2} S_{\rho\lambda} A^i_{\mu\nu} \,,
\end{aligned} \tag{2.7}$$

where $\langle \rho^r|T_{\rho\lambda}|\rho^{r'}\rangle S_{\rho\lambda} = e^r_\tau W_{\tau\sigma\lambda\rho} e^{r'}_\sigma S_{\rho\lambda}$, $W_{\tau\sigma\lambda\rho}$ is the amplitude of the ρ-γ scattering, r, r' — polarization indices (for the transversal polarization r, $r' = 1, 2$, for the longitudinal polarization r, r' have the only value).

On the other hand (see (2.2)),

$$W_{\mu\nu\rho\lambda} e^r_\mu e^r_\nu = 2\mathcal{M}_\rho p_\rho p_\lambda \,, \tag{2.8}$$

where the averaging over polarizations is assumed.

Let us discuss the tensor structure of $W_{\mu\nu\rho\lambda}$. Starting from the most general form of $W_{\mu\nu\rho\lambda}$ and using the equations $W_{\mu\nu\rho\lambda} p_\mu = W_{\mu\nu\rho\lambda} p_\nu = 0$, which follow from the current conservation, we obtain:

$$\begin{aligned}
W_{\mu\nu\rho\lambda} = a&\left(\frac{1}{p^2} p_\mu p_\nu p_\rho p_\lambda - g_{\mu\rho}(p_\nu p_\lambda - g_{\nu\lambda} p^2) - g_{\nu\lambda} p_\mu p_\rho\right) + \\
&+ b_1 (g_{\mu\nu} p_\rho p_\lambda - g_{\mu\rho}(p_\nu p_\lambda - g_{\nu\lambda} p^2) - g_{\nu\lambda} p_\mu p_\rho) + \\
&+ b_2 \left(g_{\mu\lambda}(p_\nu p_\rho - g_{\nu\rho} p^2) - g_{\mu\rho}(p_\nu p_\lambda - g_{\nu\lambda} p^2) + g_{\nu\rho} p_\mu p_\lambda - g_{\nu\lambda} p_\mu p_\rho\right) + \\
&+ b_3 g_{\lambda\rho}(p_\mu p_\nu - g_{\mu\nu} p^2) \,.
\end{aligned} \tag{2.9}$$

In order to find the invariants, related to the second moment, we specify reference system. We work in the reference, where the ρ-meson momentum has the form $p_\mu =$

$(p_0, 0, 0, p_3)$ and the ρ-meson polarization vectors are: $e^L_\mu = (e_0, 0, 0, e_3)$, $e^L_\mu e^L_\mu = -1$, $e^L_\mu p_\mu = 0$ for the longitudinal polarization and $e^{T_1}_\mu = (0, 1, 0, 0)$, $e^{T_2}_\mu = (0, 0, 1, 0)$ for the transversal polarization.

We consider the certain components of the ρ-meson momentum vector, for example, $\rho = 3$ and $\lambda = 0$. Using expression (2.9), we obtain for $W_{\mu\nu 30} e^r_\mu e^r_\nu$:

$$\begin{aligned} W_{\mu\nu 30} e^L_\mu e^L_\nu &= a p_0 p_3 \,, \\ \frac{1}{2} \sum_{i=1,2} W_{\mu\nu 30} e^{T_i}_\mu e^{T_i}_\nu &= -b_1 p_0 p_3 \,. \end{aligned} \tag{2.10}$$

Considering equation (2.8) in the same reference and choosing $\rho = 3$, $\lambda = 0$, one can see that the related to the second moment invariants are: a for the longitudinal and $-b_1$ for transversal ρ-meson polarization:

$$a = 2\mathcal{M}^L_\rho \,, \qquad -b_1 = 2\mathcal{M}^T_\rho \,, \tag{2.11}$$

where $\mathcal{M}^L_\rho$ and $\mathcal{M}^T_\rho$ are the second moments of the quark structure functions of the longitudinally and transversally polarized ρ-meson.

Substituting general form (2.9) into (2.7), we find that a is the only coefficient at the term $p_\mu p_\nu p_\rho p_\lambda S_{\rho\lambda} A^i_{\mu\nu}$ and b_1 is the only coefficient at the term $g_{\mu\nu} p_\rho p_\lambda S_{\rho\lambda} A^i_{\mu\nu}$. We come to conclusion that one should construct the projection operator $A^i_{\mu\nu}$, which cuts the structure $p_\mu p_\nu$ in the case of the longitudinal polarization and $g_{\mu\nu}$ in the case of the transversal polarization. However, practically instead of constructing the projection operators one can simply consider in $\Pi_{\mu\nu\rho\lambda}$ (2.6) the structure $p_\mu p_\nu p_\rho p_\lambda$ for the longitudinal polarization and the structure $g_{\mu\nu} p_\rho p_\lambda$ for the transversal polarization.

It is worth to note that such an analysis is not new, for example, in [4] kinematical structures were chosen in the same manner.

Now one can write down phenomenological representations of the invariants at the selected kinematical structures. Using formula (2.7) and taking into account also the contribution of the continuum, we obtain for the invariant at the structure $p_\mu p_\nu p_\rho p_\lambda$:

$$\begin{aligned} \Pi^L(p^2) &= \int ds \frac{\rho_L(s)}{(s - p^2)^2} + \cdots \,, \\ \rho_L(s) &= -\frac{m^4_\rho}{g^2_\rho} \frac{a}{s} \delta(s - m^2_\rho) + f_L(s) \theta(s - s_\rho) \,, \end{aligned} \tag{2.12}$$

and for the invariant function at the structure $g_{\mu\nu} p_\rho p_\lambda$:

$$\begin{aligned} \Pi^T(p^2) &= \int ds \frac{\rho_T(s)}{(s - p^2)^2} + \cdots \,, \\ \rho_T(s) &= -\frac{m^4_\rho}{g^2_\rho} b_1 \delta(s - m^2_\rho) + f_T(s) \theta(s - s_\rho) \,. \end{aligned} \tag{2.13}$$

In these equations dots mean the contributions of non-diagonal transitions (for example, $\langle 0|j_\mu|\rho^\star\rangle\langle\rho^\star|T_{\rho\lambda}S_{\rho\lambda}|\rho\rangle\langle\rho|j_\nu|0\rangle$, where $\rho^\star$ is the excited state with the same quantum numbers as ρ), functions f_L and f_T represent continuum contribution and s_ρ is the continuum threshold for the ρ-meson.

Taking into account relations (2.11) and retaining only the terms, which do not vanish after Borel transformation, we obtain:

$$\Pi^L(p^2) = -\frac{m_\rho^2}{g_\rho^2}\frac{2\mathcal{M}_\rho^L}{(m_\rho^2-p^2)^2} + \frac{\tilde{C}_L}{m_\rho^2-p^2} + \int\limits_{s_\rho}^{\infty} ds \frac{f_L(s)}{(s-p^2)^2}\,, \tag{2.14}$$

$$\Pi^T(p^2) = \frac{m_\rho^4}{g_\rho^2}\frac{2\mathcal{M}_\rho^T}{(m_\rho^2-p^2)^2} + \frac{\tilde{C}_T}{m_\rho^2-p^2} + \int\limits_{s_\rho}^{\infty} ds \frac{f_T(s)}{(s-p^2)^2}\,. \tag{2.15}$$

Here $\mathcal{M}_\rho^L$ and $\mathcal{M}_\rho^T$ are the second moments of the quark structure functions of the longitudinally and transversally polarized ρ-meson, $\tilde{C}_L$, $\tilde{C}_T$ appear due to non-diagonal transitions.

In the next section functions $\Pi^L(p^2)$ and $\Pi^T(p^2)$ in the left-hand sides of (2.14), (2.15) will be calculated as the operator product expansion series.

3. Calculation of the vector current correlator

Let us calculate in the correlator (2.4) the functions at the selected kinematical structures $p_\mu p_\nu p_\rho p_\lambda$ and $g_{\mu\nu}p_\rho p_\lambda$, $\Pi^L(p^2)$ and $\Pi^T(p^2)$ correspondingly, basing on the operator product expansion in QCD.

First of all, we write down the quark propagator in the external field $S_{\mu\nu}$ [7] (figure 1):

$$\langle Tq_\alpha^a(x)\overline{q}_\beta^b(0)\rangle_S = \frac{i\delta^{ab}}{2\pi^2x^4}\left(\hat{x} + S_{\rho\lambda}\left(x_\rho\gamma_\lambda - \frac{4x_\rho x_\lambda}{x^2}\hat{x}\right)\right)_{\alpha\beta}.$$

Here α, β are spinor indices, a, b-color indices.

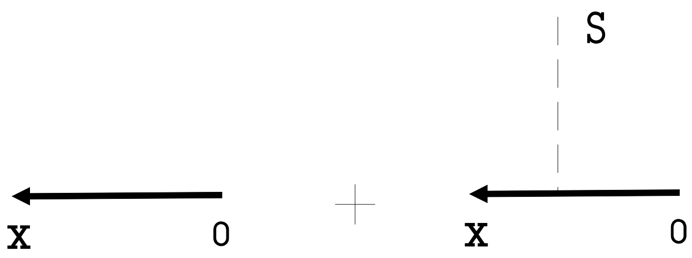

Figure 1: Diagrams for the quark propagator (solid line) in the external tensor field (dashed line).

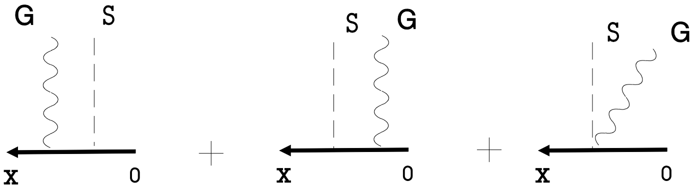

Figure 2: Diagrams for the quark propagator (solid line) in the external tensor field (dashed line) and soft gluon field (wave line).

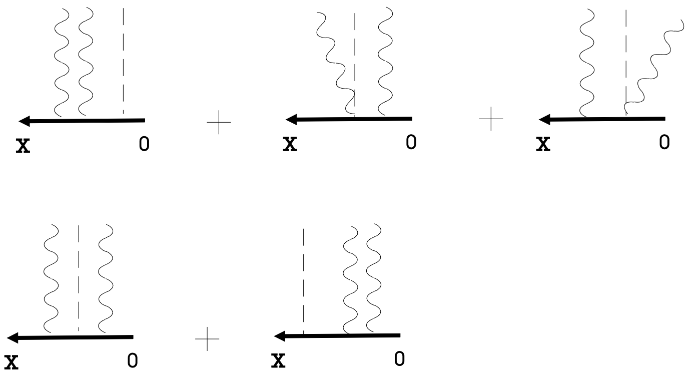

Figure 3: Diagrams for the quark propagator in the external tensor field and soft gluon field. All notations are as in figure 2.

An interaction with the soft gluon field modifies this expression[1] (we write down the proportional to $S_{\rho\lambda}$ terms only):

$$\langle T q^a_\alpha(x)\overline{q}^b_\beta(0)\rangle_{GS} = \frac{gG^n_{\eta\sigma}(t^n)^{ab}S_{\rho\lambda}}{8\pi^2x^2}\Bigg(\frac{x_\rho x_\lambda \gamma_\eta \hat{x}\gamma_\sigma}{x^2} + \frac{x_\rho}{4}(\gamma_\lambda\gamma_\eta\gamma_\sigma - \gamma_\sigma\gamma_\eta\gamma_\lambda) + \\ + \frac{g_{\lambda\sigma}}{2}(\gamma_\eta\hat{x}\gamma_\rho - \gamma_\rho\hat{x}\gamma_\eta)\Bigg)_{\alpha\beta}$$

(all propagators in figure 2),

$$\langle T q^a_\alpha(x)\overline{q}^b_\beta(0)\rangle_{SGG} = \frac{g^2\langle G^2\rangle\delta^{ab}S_{\rho\lambda}}{2^6 3^2\pi^4}\int d^4z \frac{z_\rho}{(x-z)^4 z^2}\Big((\hat{x}-\hat{z})(\hat{z}\gamma_\lambda - z_\lambda)\Big)_{\alpha\beta}$$

(all propagators in figure 3).

Let us begin with the case of the longitudinal polarization.

We consider only the operators, which contribute to $\Pi^L(p^2)$ at the structure $p_\mu p_\nu p_\rho p_\lambda$, and retain only the terms, remained after Borel transformation.

The contribution of the loop diagrams (figure 4) to $\Pi^L(p^2)$ is equal to

$$-\frac{1}{2\pi^2}\int\limits_0^\infty \frac{ds}{(s-p^2)^2}\,.$$

[1]The proportional to $x_\eta x_\sigma x_\tau$ parts of these expressions can be found in [7].

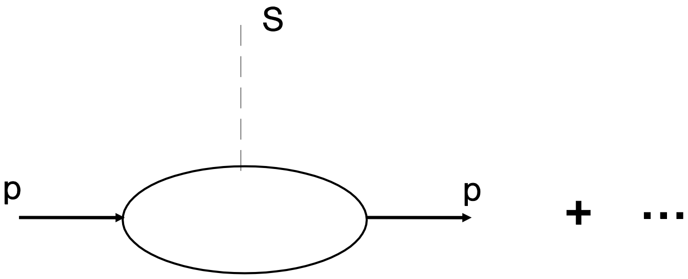

Figure 4: Loop diagrams in the external tensor field. p represents the momentum of the current. Other notations are as in figure 1.

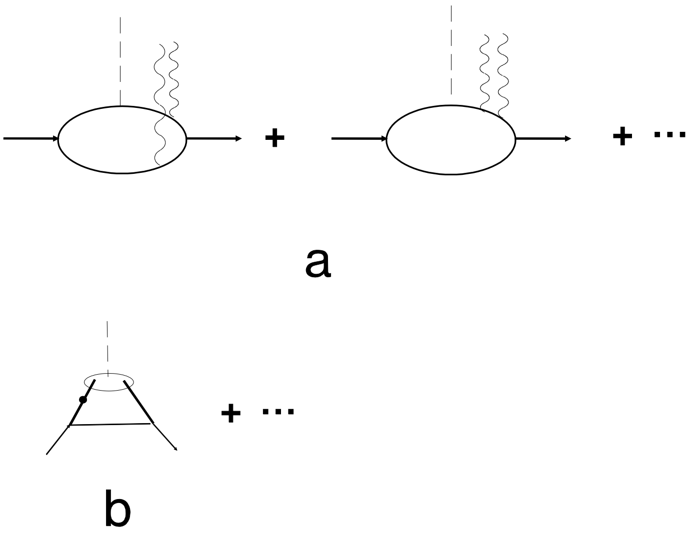

Figure 5: Diagrams of dimension 4 operators. Black circle on the quark line means derivative. Dots stand for permutations. Other notations are as in figure 2.

According to the quark-hadron duality, the continuum contribution in the interval of $P^2 = -p^2$ from s_ρ to infinity is determined by the bare loop in this interval. Therefore, function f_L in (2.14) is constant: $f_L = -1/(2\pi^2)$.

Diagrams, related to the operator $G^n_{\mu\nu}G^n_{\mu\nu}$, are shown in figure 5a. The field induced vacuum expectation value (figure 5b) does not contribute to this kinematical structure.

There are a number of vacuum expectation values of dimension 6 operators (see figure 6a). Some of them are not related to the external field. They are:

$$\begin{aligned}
\langle \overline{q}^a_\alpha q^b_\beta D_\tau G^n_{\eta\sigma}\rangle &= \frac{-g\langle\overline{q}q\rangle^2}{3^3 2^4}(t^n)^{ba}(g_{\tau\sigma}\gamma_\eta - g_{\tau\eta}\gamma_\sigma)_{\beta\alpha}\,,\\
\langle \overline{q}^a_\alpha (D_\tau q^b_\beta) G^n_{\eta\sigma}\rangle &= \frac{-g\langle\overline{q}q\rangle^2}{3^3 2^5}(t^n)^{ba}(g_{\tau\eta}\gamma_\sigma - g_{\tau\sigma}\gamma_\eta - i\epsilon_{\tau\eta\sigma\xi}\gamma_5\gamma_\xi)_{\beta\alpha}\,,\\
\langle \overline{q}^a_\alpha D_\sigma D_\eta D_\tau q^b_\beta\rangle &= \frac{-ig^2\langle\overline{q}q\rangle^2}{3^5 2^4}\delta^{ba}(g_{\eta\tau}\gamma_\sigma + g_{\sigma\eta}\gamma_\tau - 5g_{\sigma\tau}\gamma_\eta)_{\beta\alpha}\,.
\end{aligned}$$

The derivation of these expressions can be found in [9].

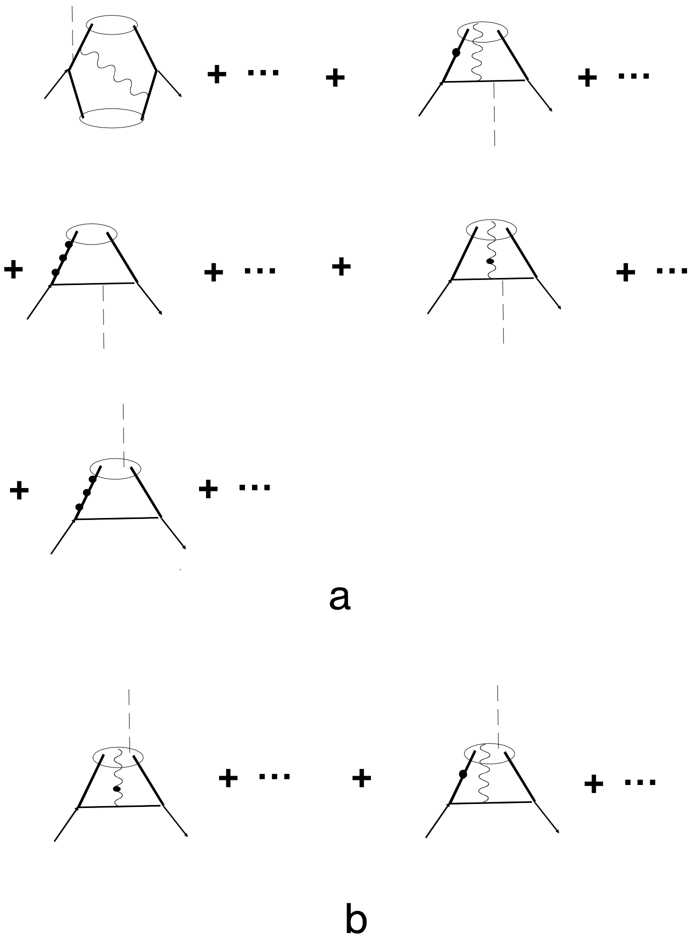

Figure 6: Diagrams of dimension 6 operators. Black circles on the quark or gluon lines mean derivatives. All other notations are as in the previous figures.

The only field induced vacuum expectation value, which contributes to $\Pi^L(p^2)$, has the form (the last diagram in figure 6a):

$$\langle (D_{\{\sigma} D_\tau D_{\eta\}} q^a_\alpha(0)) \overline{q}^b_\beta(0)\rangle_S = \delta^{ab}(c_1 S_{\{\sigma\tau}\gamma_{\eta\}} + c_2 g_{\{\sigma\tau} S_{\eta\}\chi}\gamma_\chi)_{\alpha\beta}\,.$$

The factors c_1 and c_2 are determined in appendix. Only factor c_1 is involved in $\Pi^L(p^2)$. According to $(A12)$, $(A13)$,

$$c_1 = -3k(1+\epsilon_1)\,, \qquad k = -\frac{ig^2\langle\overline{q}q\rangle^2}{3^5 2^4}\,.$$

Here

$$\epsilon_1 = \frac{ig}{3k}\left(-d_1 + \frac{d_3}{3}\right)$$

represents the contribution of nonfactorizable vacuum expectation values into c_1. In appendix the set of unknown parameters d_1, d_3, d_4 is introduced and it is shown that all possible nonfactorizable field induced vacuum expectation values of dimension 6 operators can be expressed in terms of these three parameters.

For d_1, d_3, d_4 the following relation takes place ($A11$):

$$\frac{4}{3}d_1 - d_3 + d_4 = -\frac{g\langle \overline{q}q\rangle^2}{3^3 2^4}\,. \tag{3.1}$$

In assumption that the absolute value of each of these parameters is of the order of $g\langle \overline{q}q\rangle^2/3^3 2^4$ (or less), one easily see that $|\epsilon_1|$ can reach the values up to $1 \div 2$, i.e. the accuracy of the estimation of c_1 is low — about 100-200%.

Fortunately, the contribution of the related to $\langle (D_{\{\sigma} D_\tau D_{\eta\}} q^a_\alpha(0))\overline{q}^b_\beta(0)\rangle_S$ diagram to the total correction of the dimension 6 operators is strongly numerically suppressed (the diagrams with the hard gluon exchange are dominating). Quantitative analysis shows that large $|\epsilon_1| \approx 1 \div 2$ results in 30% uncertainty in the total correction of the dimension 6 operators for any more or less reasonable values of d_1, d_3, d_4. As we shall see, the operators of dimension 6 give less than 5% of the value of the second moment. Therefore, even 100% uncertainty in ϵ_1 does not influence the accuracy of our results, and we can safely neglect it.

Collecting all the terms, we obtain for $\Pi^L(p^2)$:

$$\Pi^L(p^2) = -\frac{1}{2\pi^2}\int\limits_0^\infty \frac{ds}{(s-p^2)^2} + \frac{1}{9}\frac{\langle(\alpha_s/\pi)G^2\rangle}{p^6} + \frac{56}{27}\frac{g^2\langle\overline{q}q\rangle^2}{p^8}\,. \tag{3.2}$$

Now we consider the case of the transversally polarized ρ-meson.

Again, we take into account only the operators, which contribute to the function $\Pi^T(p^2)$ at the structure $g_{\mu\nu}p_\rho p_\lambda$, and omit the terms, vanished after Borel transformation.

The loop diagrams (figure 4) give to $\Pi^T(p^2)$ the following contribution:

$$\frac{1}{2\pi^2}\int\limits_0^\infty \frac{s\,ds}{(s-p^2)^2}\,. \tag{3.3}$$

Therefore, function f_T in (2.15) is: $f_T(s) = s/(2\pi^2)$.

The diagrams with the gluon condensate (dimension 4) are shown in figure 5a. The contribution of these diagrams to $\Pi^T(p^2)$ has an infrared divergence in the chiral limit. To calculate it, we introduce nonzero mass μ_q for the quark, interacting with the external field $S_{\rho\lambda}$. We have:

$$-\frac{2}{9p^4}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left(\ln\left(\frac{-p^2}{\mu_q^2}\right)+\frac{1}{3}\right). \tag{3.4}$$

However, in the case of the transversal polarization one more vacuum expectation value of dimension 4 operator in the external field appears: $\langle (D_\sigma q^a_\alpha(0))\overline{q}^b_\beta(0)\rangle_S$ (figure 5b). Its most general form is:

$$\langle (D_\sigma q^a_\alpha(0))\overline{q}^b_\beta(0)\rangle_S = c_3 S_{\sigma\tau}(\gamma_\tau)_{\alpha\beta}\delta^{ab}\,. \tag{3.5}$$

Factor c_3 was found in the paper [10].

According to [10], in order to obtain c_3, one should construct a sum rule for the correlator

$$\Pi_{\mu\nu\sigma\tau}(p) = i\int d^4x\, e^{ipx}\langle T(j_{\mu\nu}(x) j^+_{\sigma\tau}(0)\rangle\,,$$
$$j_{\mu\nu} = \frac{i}{2}\sum_{\psi=u,d}\overline{\psi}\left(\overrightarrow{D}_{\{\mu}\gamma_{\nu\}} - \overleftarrow{D}_{\{\mu}\gamma_{\nu\}}\right)\psi\,. \tag{3.6}$$

The phenomenological part of the sum rule is saturated by the contributions of the $f_2(1270)$ meson and continuum. In the operator product expansion part one should subtract not only perturbative contribution (3.3), but also the contribution of the operator $G^n_{\mu\nu}G^n_{\mu\nu}$, since it has been taken into account in (3.4). In such a way it was obtained [10]:

$$c_3 = \frac{i}{12}\left(\frac{3s_f^2}{320\pi^2} - g_f^2 m_f^4 - \frac{1}{18}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left(\ln\frac{s_f}{\mu_q^2} - \frac{1}{2}\right)\right)\,. \tag{3.7}$$

Here $m_f = 1.27\,\mathrm{GeV}$ is the f_2-meson mass, s_f is the continuum threshold for the f_2-meson and g_f is the coupling constant of the f_2-meson with tensor current. Using (3.7), we can calculate the diagrams in figure 5b.

The total contribution of the dimension 4 operators to $\Pi^T(p^2)$ is equal to

$$-\frac{1}{p^4}\left(\frac{3s_f^2}{80\pi^2} - 4g_f^2 m_f^4 + \frac{2}{9}\langle\frac{\alpha_s}{\pi}G^2\rangle\left(\ln\left(\frac{-p^2}{s_f}\right) + \frac{5}{6}\right)\right)\,.$$

It should be noted that this expression does not contain the quark mass μ_q, i.e. the infrared divergences cancel out.

The operators of dimension 6 were discussed above (figure 6a). Besides them, new vacuum expectation values of the operators of the same dimension appear (figure 6b):

$$\langle\overline{q}^a_\alpha G^n_{\mu\nu}D_\eta q^b_\beta\rangle_S\,,\qquad \langle\overline{q}^a_\alpha\overleftarrow{D_\eta}G^n_{\mu\nu}q^b_\beta\rangle_S\,,\qquad \langle\overline{q}^a_\alpha(D_\eta G^n_{\mu\nu})q^b_\beta\rangle_S\,. \tag{3.8}$$

All of them are expressed in terms of parameters d_1, d_3, d_4 (see appendix).

The contribution of vacuum expectation values (3.8) is suppressed numerically even more than in the case of the longitudinal polarization (by the factor about 1/30). Using the same estimation for d_1, d_3, d_4 as in the previous case (see (3.1)), one can find that the uncertainty in the value of the total contribution of dimension 6 operators, which appears due to these parameters, is less than 15%. This gives the uncertainty about few percents only in the final answer for the second moment. Therefore, one can neglect all the terms, which contain d_1, d_3, d_4.

It should be noted here that the field induced vacuum expectation value $\langle\overline{q}^a_\alpha\overline{q}^b_\beta q^c_\gamma q^d_\delta\rangle_S$ is zero (It can be shown with the help of the factorization hypothesis, see appendix).

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As the result we obtain for the operator product expansion part of the sum rule for the transversally polarized ρ-meson:

$$\Pi^T(p^2) = \frac{1}{2\pi^2}\int\limits_0^\infty \frac{sds}{(s-p^2)^2} - $$
$$-\frac{1}{p^4}\left(\frac{3s_f^2}{80\pi^2} - 4g_f^2m_f^4 + \frac{2}{9}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left(\ln\left(\frac{-p^2}{s_f}\right)+\frac{5}{6}\right)\right) - \frac{140}{81}\frac{g^2\langle\overline{q}q\rangle^2}{p^6}\,. \quad (3.9)$$

4. Results and discussion

To complete the calculations, we apply to the phenomenological (2.14), (2.15) and operator product expansion (3.2), (3.9) parts of the sum rules the Borel transformation $\hat{B}(M^2)$,

$$\hat{B}(M^2) = \lim_{\substack{P^2,n\to\infty \\ P^2/n=M^2}} \frac{(P^2)^{n+1}}{n!}\left(-\frac{d}{dP^2}\right)^n ,$$

where $P^2 = -p^2 > 0$, and equate them. We obtain for the longitudinal polarization:

$$\frac{g_\rho^2}{m_\rho^2}e^{m_\rho^2/M^2}\left(\frac{M^2}{4\pi^2}\left(1-e^{-s_\rho/M^2}\right)+\frac{1}{36}\frac{\langle(\alpha_s/\pi)G^2\rangle}{M^2}-\frac{14}{81}\frac{g^2\langle\overline{q}q\rangle^2}{M^4}\right) = \mathcal{M}_\rho^L(M^2)+C_LM^2\,, \quad (4.1)$$

and for the transversal polarization:

$$\frac{g_\rho^2}{m_\rho^4}e^{m_\rho^2/M^2}\left(\frac{M^4}{4\pi^2}\left(1-e^{-s_\rho/M^2}\left(1+\frac{s_\rho}{M^2}\right)\right)-\frac{3s_f^2}{160\pi^2}+2g_f^2m_f^4 - \right. \quad (4.2)$$
$$\left. -\frac{1}{9}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left(\ln\frac{M^2}{s_f}+\frac{11}{6}-C_E\right)+\frac{35}{81}\frac{g^2\langle\overline{q}q\rangle^2}{M^2}\right) = \mathcal{M}_\rho^T(M^2)+C_TM^2.$$

Here $C_E = 0.577\ldots$ is the Euler constant, C_L, C_T appear due to the nondiagonal transitions.

Generally speaking, the second moments in these equations are functions of the squared Borel mass M^2 too. To analyze these sum rules we calculate the second moments at arbitrary fixed point μ. We take into account that the following linear combinations of the second moments of the quark $\mathcal{M}_\rho$ and gluon $\mathcal{M}_{\rho G}$ structure functions are renormcovariant [7]:

$$\left(\frac{16}{25}\mathcal{M}_\rho(M^2)-\frac{9}{25}\mathcal{M}_{\rho G}(M^2)\right)L^{50/81} = \frac{16}{25}\mathcal{M}_\rho(\mu^2)-\frac{9}{25}\mathcal{M}_{\rho G}(\mu^2)\,, \quad (4.3)$$
$$\mathcal{M}_\rho(M^2)+\mathcal{M}_{\rho G}(M^2) = \mathcal{M}_\rho(\mu^2)+\mathcal{M}_{\rho G}(\mu^2) = 1\,. \quad (4.4)$$

Here

$$L = \frac{\ln(M^2/\Lambda_{QCD}^2)}{\ln(\mu^2/\Lambda_{QCD}^2)}\,,$$

μ is the operator expansion normalization point.

From (4.3), (4.4) one can express $\mathcal{M}_\rho(M^2)$ and $\mathcal{M}_{\rho G}(M^2)$ in terms of $\mathcal{M}_\rho(\mu^2)$ and $\mathcal{M}_{\rho G}(\mu^2)$ and substitute them into (4.1), (4.2). Excluding $\mathcal{M}_{\rho G}(\mu^2)$ with the help of (4.4), we obtain the following sum rules.

For the longitudinally polarized ρ-meson:

$$\frac{9}{25}(1-L^{50/81})+\frac{g_\rho^2}{m_\rho^2}e^{m_\rho^2/M^2}L^{50/81}\left(\frac{M^2}{4\pi^2}\left(1-e^{-s_\rho/M^2}\right)+\right.$$
$$\left.+\frac{1}{36}\frac{\langle(\alpha_s/\pi)G^2\rangle}{M^2}-\frac{14}{81}\frac{g^2\langle\overline{q}q\rangle^2}{M^4}\right)=\mathcal{M}_\rho^L(\mu^2)+C_L M^2\,. \tag{4.5}$$

For the transversally polarized ρ-meson:

$$\frac{9}{25}(1-L^{50/81})+\frac{g_\rho^2}{m_\rho^4}e^{m_\rho^2/M^2}L^{50/81}\left(\frac{M^4}{4\pi^2}\left(1-e^{-s_\rho/M^2}\left(1+\frac{s_\rho}{M^2}\right)\right)-\right.$$
$$\left.-\frac{3s_f^2}{160\pi^2}+2g_f^2m_f^4-\frac{1}{9}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\left(\ln\frac{M^2}{s_f}+\frac{11}{6}-C_E\right)+\frac{35}{81}\frac{g^2\langle\overline{q}q\rangle^2}{M^2}\right)=$$
$$=\mathcal{M}_\rho^T(\mu^2)+C_T M^2\,. \tag{4.6}$$

We use the following values of parameters:

- $\mu = 1\,\mathrm{GeV}$ — the operator expansion normalization point,
- $\Lambda_{QCD} = 0.2\,\mathrm{GeV}$,
- $m_\rho = 0.77\,\mathrm{GeV}$ — the ρ-meson mass,
- $g_\rho^2/(4\pi) = 1.27$ — the ρ-γ coupling constant,
- $s_\rho = 1.5\,\mathrm{GeV}^2$ — the continuum threshold for ρ-meson [1], $\langle(\alpha_s/\pi)G^2\rangle = 0.012$ GeV^4 — the gluon condensate [1],
- $g^2\langle\overline{q}q\rangle^2 = 0.23\times10^{-2}\,\mathrm{GeV}^6$ — the quark condensate [1],
- $m_f = 1.27\,\mathrm{GeV}$ — the f_2-meson mass.

The parameters m_f, s_f and g_f appear in sum rule for the transversal polarization (4.6), because we consider the correlator of the tensor currents (3.6) to calculate the nonfactorizible vacuum expectation value $\langle(D_\sigma q_\alpha^a(0))\overline{q}_\beta^b(0)\rangle_S$. This correlator was considered in [10], when the following sum rule was obtained:

$$\frac{1}{\pi M^2}\int \mathrm{Im}\,\Pi(s)e^{-s/M^2}ds=\frac{3}{80\pi^2}M^4-\frac{1}{18}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle+\frac{1}{9}\frac{g^2\langle\overline{q}q\rangle^2}{M^2}\,,$$
$$\mathrm{Im}\,\Pi(s)=\pi m_f^6 g_f^2\delta(s-m_f^2)+\frac{3s^2}{160\pi}\Theta(s-s_f)\,. \tag{4.7}$$

So, one can exclude g_f from the sum rule (4.6) by direct substituting it from (4.7). The continuum threshold for the f_2-meson s_f can be found from the sum rule (4.7)

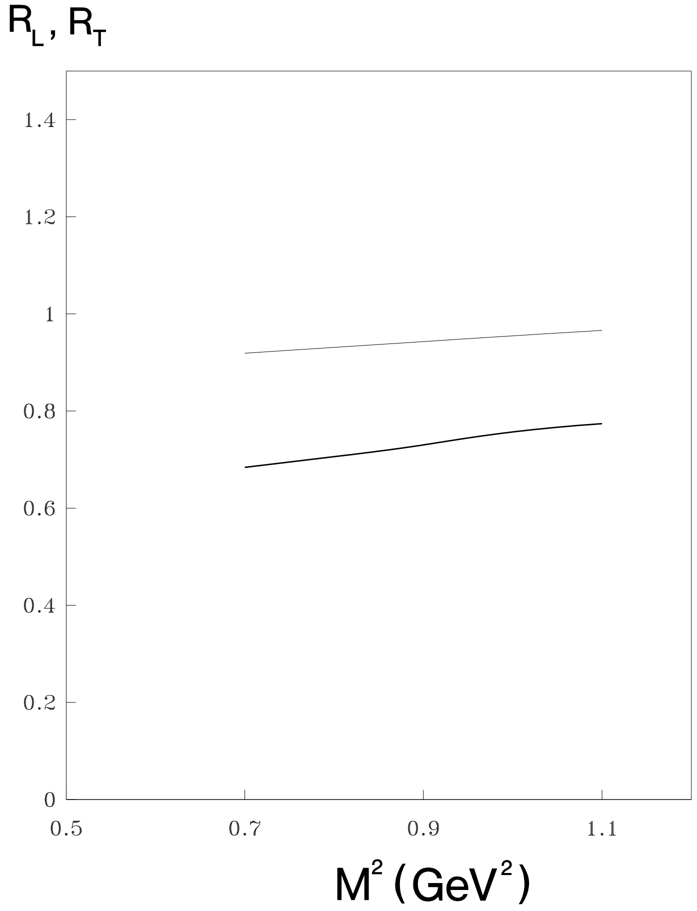

Figure 7: The left hand sides of equations (4.5) R_L (the thin curve), (4.6) R_T (the thick curve) as functions of squared Borel mass M^2.

for the f_2-meson mass. An analysis shows that $s_f \approx 2.5 \div 2.7\,\mathrm{GeV}^2$. For the further calculation we choose $s_f = 2.6\,\mathrm{GeV}^2$, the corresponding value of the coupling constant of the f_2-meson with tensor current is: $g_f = 0.044$.

One can extract the values of $\mathcal{M}_\rho^L(\mu^2)$, $\mathcal{M}_\rho^T(\mu^2)$ in two ways. We can approximate the left-hand sides of the equations (4.5), (4.6) by a straight line in the certain interval of M^2 and find it ordinate at zero Borel mass. The left-hand sides of (4.5), (4.6) as functions of M^2 are shown in figure 7. Another way consists in applying of the operator $1 - M^2(d/dM^2)$ to equations (4.5), (4.6).

Both these ways lead to the same values:

$$\mathcal{M}_\rho^L = 0.84\,, \qquad \mathcal{M}_\rho^T = 0.5\,. \tag{4.8}$$

The sum rule (4.5) for the longitudinal polarization gives the value of the second moment $\mathcal{M}_\rho^L$ with a very good accuracy. The left-hand side of (4.5) as a function of M^2 in the interval $0.7\,\mathrm{GeV}^2 \leq M^2 \leq 1.1\,\mathrm{GeV}^2$ is close to horizontal straight line (the thin curve in figure 7). This means that the contribution of nondiagonal terms is very small.

The contributions of the dimension 4 and 6 operators give not more than 5% of the total answer each.

The accuracy in (4.8) is determined by the uncertainties in the values of the quark and gluon condensates. The results (4.8) correspond to $g^2\langle\overline{q}q\rangle^2 = 0.23 \times 10^{-2}\,\mathrm{GeV}^6$ [1]. However, from the analysis of τ-decay data [12] the larger value of the quark condensate was obtained recently:

$$g^2\langle\overline{q}q\rangle^2 = (0.38 \pm 0.12) \times 10^{-2}\,\mathrm{GeV}^6\,. \tag{4.9}$$

These results do not contradict one another within the errors, especially if one takes into account that (4.9) was obtained from the sum rule, in which the α_s-corrections were accounted, whereas in [1] they were not. Nevertheless, the difference between the average values of the quark condensate is about 50%.

The uncertainty in the value of the gluon condensate is also about 50%.

Taking into account all uncertainties discussed, one can find that the accuracy of the value of the second moment in the case of the longitudinal polarization is not worse than $5 \div 10\%$.

A variation of the continuum threshold for the ρ-meson s_ρ in the reasonable limits does not affect the result.

In the sum rule for the transversal polarization (4.6) the same interval $0.7\,\mathrm{GeV}^2 \leq M^2 \leq 1.1\,\mathrm{GeV}^2$ is investigated. The contribution of the dimension 4 operators gives not more than 40% of the total value, the contribution of the dimension 6 operators — not more than 20%. These contributions have opposite signs, at $M^2 = 0.7\,\mathrm{GeV}^2$ the last contribution is equal approximately to one half of the first one and reduces with increase of M^2.

The increase of the value of the gluon condensate by the factor 1.5 results in not more than 10% rise of the value of the second moment, the changing of the value of the quark condensate from $g^2\langle\overline{q}q\rangle^2 = 0.23\times 10^{-2}\,\mathrm{GeV}^6$ to $g^2\langle\overline{q}q\rangle^2 = 0.38\times 10^{-2}\,\mathrm{GeV}^6$ increases $\mathcal{M}_\rho^T$ by $15 \div 20\%$.

A variation of s_f in the interval $2.5 - 2.7\,\mathrm{GeV}^2$ changes the value of the second moment $\mathcal{M}_\rho^T$ by $\pm 10\%$.

One should evaluate also the uncertainties, which appear due to approximation procedure. In [13] it was shown that nonlinear terms can be safely neglected, when $\mathcal{M}_\rho^i \ll C_i M^2$, $i = L, T$. This condition takes place in the case of the longitudinal polarization. In the transversal polarization case $\mathcal{M}_\rho^T$ is only two times as large as $C_T M^2$ in the considered interval, and so we use chi-square technique to estimate deviations from linearity. Denoting the left hand side of equation (4.6) as $R_T(M^2)$, we find:

$$\delta = \frac{1}{\mathcal{M}_\rho^T}\left(\sum_{j=1}^{5}(R_T(M_j^2) - \mathcal{M}_\rho^T - C_T M_j^2)^2\right)^{1/2} \approx 0.003\,,$$

where $M_1^2 = 0.7\,\mathrm{GeV}^2$, $M_2^2 = 0.8\,\mathrm{GeV}^2$, $\ldots$, $M_5^2 = 1.1\,\mathrm{GeV}^2$. Since $\delta \ll 1$, the nonlinear terms can be neglected also in the case of the transversal polarization.

Collecting all these uncertainties, one can find that the accuracy of $\mathcal{M}_\rho^T$ is about 20%.

Our final results are:

$$\mathcal{M}_\rho^L = 0.84 \pm 0.08\,, \tag{4.10}$$

$$\mathcal{M}_\rho^T = 0.5 \pm 0.1\,. \tag{4.11}$$

Now one can calculate the second moment $\mathcal{M}_\rho$ of the non-polarized ρ-meson:

$$\mathcal{M}_\rho = \frac{1}{3}\mathcal{M}_\rho^L + \frac{2}{3}\mathcal{M}_\rho^T\,.$$

This gives

$$\mathcal{M}_\rho = 0.60 \pm 0.13\,. \tag{4.12}$$

Results (4.10), (4.11), (4.12) agree with the values obtained from the lattice simulations[2] [14]. It should be emphasized that the large difference between $\mathcal{M}_\rho^L$ and $\mathcal{M}_\rho^T$ (about 0.34, see (4.10), (4.11)) is close to those found in [14] (about 0.29).

In [3, 4] in the framework of QCD sum rules the valence quark structure functions were calculated at the intermediate values of the Bjorken variable x. The sum rules had the resembling structure, namely, in the case of the longitudinal polarization the contributions of the quark and gluon condensates were quite small, whereas in the case of the transversal polarization the dimension 4 operators play an important role.

The large value of $\mathcal{M}_\rho^L$ (4.10) lead to the conclusion that the gluon sea in the longitudinally polarized ρ-meson is strongly suppressed (it is less than 25%, while usually gluons carry about 50% of the total momentum). So, it seems reasonable that the quark sea in the longitudinally polarized ρ-meson is suppressed also. Therefore, we can compare our result for $\mathcal{M}_\rho^L$ with the value 0.78, estimated in [4]. Let us remind that in [4] the valence quark structure functions was calculated in the interval $0.1 < x < 0.75$, whereas at small x Regge asymptotic and at large x quark counting rules were assumed. The fact that our result is quite close to the value of [4] verifies these assumptions. Thus, now the quark structure function can be described in the whole region $0 < x < 1$.

Unfortunately, the same analysis for the transversal polarization can not be done, mainly because of the fact that in [4] the valence quarks were considered, whereas in the present paper we can not separate valence and sea quarks.

The second moment of the quark structure function of the pion $\mathcal{M}_\pi$ was calculated in [7] and [8]. However, the contributions of the dimension 6 operators in these papers differ from each other. That is why we recalculate $\mathcal{M}_\pi$.

[2] In order to compare the results, one should take into account a difference in choice of μ^2.

$\mathcal{M}_\pi$ is related to the tensor $T_{\mu\nu}$ (2.1) in the similar way:

$$\langle\pi|T_{\mu\nu}|\pi\rangle = 2\mathcal{M}_\pi p_\mu p_\nu \,,$$

p_μ is the pion momentum.

Following [7], we consider the correlator of the axial currents $j_\mu = \overline{u}\gamma_\mu\gamma_5 d$

$$\Pi_{\mu\nu}(p) = i\int d^4x\, e^{ipx}\langle T(j_\mu(x) j_\nu^+(0))\rangle_S$$

and saturate its dispersion relation by the contributions of the pion and continuum. We obtain the following sum rule:

$$\frac{9}{25}(1-L^{50/81}) + \frac{1}{f_\pi^2}L^{50/81}\left(\frac{M^2}{4\pi^2}\left(1-e^{-s_\pi/M^2}\right) + \frac{1}{36}\frac{\langle(\alpha_s/\pi)G^2\rangle}{M^2} + \frac{34}{81}\frac{g^2\langle\overline{q}q\rangle^2}{M^4}\right) = \\ = \mathcal{M}_\pi(\mu^2) + C_\pi M^2 \,. \qquad (4.13)$$

In this equation $f_\pi = 0.13\,\mathrm{GeV}$, $s_\pi = 0.9\,\mathrm{GeV}$ is the continuum threshold for the pion and C_π appears due to the non-diagonal transitions. From (4.9) we obtain:

$$\mathcal{M}_\pi = 0.6 \pm 0.1 \,. \qquad (4.14)$$

The contribution of the dimension 6 operators in (4.13) is very close[3] to the corresponding term in [8] and strongly differs from [7]. The numerical value of $\mathcal{M}_\pi$ (4.14) is in a good agreement with the experimental data [15] (where the second moment of the valence quark structure function was obtained) and the results of lattice calculations [14].

In [4] the value of the second moment of the valence quark structure function of the pion was estimated: 0.44. It was found also that at the intermediate values of the Bjorken variable the valence quark structure functions of the pion and non-polarized ρ-meson are close to one another. In the present paper we find that the second moments of the (valence plus sea) quark structure functions of the pion and non-polarized ρ-meson coincide (see (4.12), (4.14)). Therefore, one can expect that their quark structure functions are close in the whole interval of the Bjorken variable.

It should be noted also that the second moments of the quark structure function of the pion and transversally polarized ρ-meson are also quite close to one another (see (4.11) and (4.14)).

5. Conclusion

We see that $\mathcal{M}_\rho^L > \mathcal{M}_\rho^T$ and the difference between them certainly exceeds the accuracy of the results. So, polarization significantly influences the momentum distribution in the ρ-meson.

[3] The small difference can appear because of different ways of factorization.

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The value of the second moment in the case of the longitudinal polarization (4.10) allows one to determine the part of the momentum, carried by the gluons: $\mathcal{M}^L_{\rho G} \sim 0.2$. Usually (in the pion or nucleon or transversally polarized ρ-meson) gluons carry about one half of the total hadron momentum. It is for the first time that so small part of the momentum, carried by the gluons, was obtained.

The estimations of the second moments in [4] are in consistent with our results, whereas the techniques of the calculations have nothing in common. This fact confirms our present results and, in general, points out to the selfconsistency of the QCD sum rule approach.

The values of the second moments of quark structure functions of the ρ-meson agree with the results of lattice calculations.

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A. Some vacuum expectation values of dimension 6 operators in the external tensor field

Here we consider the following vacuum expectation values of dimension 6 operators in the external field $S_{\rho\lambda}$:

$$\langle \overline{q}^a_\alpha \hat{G}_{\mu\nu} D_\eta q^b_\beta \rangle_S\,, \qquad \langle \overline{q}^a_\alpha \overleftarrow{D_\eta} \hat{G}_{\mu\nu} q^b_\beta \rangle_S\,, \qquad \langle \overline{q}^a_\alpha (D_\eta \hat{G}_{\mu\nu}) q^b_\beta \rangle_S\,, \tag{A.1}$$

$$\langle \overline{q}^a_\alpha D_{\{\sigma} D_\tau D_{\eta\}} q^b_\beta \rangle_S\,, \tag{A.2}$$

where $\hat{G}_{\mu\nu} = G^n_{\mu\nu} t^n$, the curly brackets mean symmetrization over the Lorenz indices and the round brackets in $(D_\eta \hat{G}_{\mu\nu})$ indicate that derivative acts only on $\hat{G}_{\mu\nu}$.

Let us start with the derivation of some useful for the further analysis relations.

Using C-parity, one can easily see that[4]

$$\langle \overline{q}^a \gamma_\sigma \hat{G}_{\mu\nu} D_\eta q^b \rangle_S = \frac{1}{2}\Big(\langle \overline{q}^a \gamma_\sigma \hat{G}_{\mu\nu} D_\eta q^b \rangle_S + \langle \overline{q}^a \overleftarrow{D_\eta} \gamma_\sigma \hat{G}_{\mu\nu} q^b \rangle_S \Big) \tag{A.3}$$

and

$$\langle \overline{q}^a \gamma_5 \gamma_\sigma \hat{G}_{\mu\nu} D_\eta q^b \rangle_S = \frac{1}{2}\Big(\langle \overline{q}^a \gamma_5 \gamma_\sigma \hat{G}_{\mu\nu} D_\eta q^b \rangle_S - \langle \overline{q}^a \overleftarrow{D_\eta} \gamma_5 \gamma_\sigma \hat{G}_{\mu\nu} q^b \rangle_S \Big)\,. \tag{A.4}$$

[4]Hereafter we omit spinor and color indices, if the summation over them is assumed.

From the other side, with the accuracy up to full derivative one can write:

$$\langle\overline{q}^a\gamma_\sigma\hat{G}_{\mu\nu}D_\eta q^b\rangle_S = -\langle\overline{q}^a\overleftarrow{D_\eta}\gamma_\sigma\hat{G}_{\mu\nu}q^b\rangle_S - \langle\overline{q}^a\gamma_\sigma(D_\eta\hat{G}_{\mu\nu})q^b\rangle_S\,, \tag{A.5}$$

$$\langle\overline{q}^a\gamma_5\gamma_\sigma\hat{G}_{\mu\nu}D_\eta q^b\rangle_S = -\langle\overline{q}^a\overleftarrow{D_\eta}\gamma_5\gamma_\sigma\hat{G}_{\mu\nu}q^b\rangle_S - \langle\overline{q}^a\gamma_5\gamma_\sigma(D_\eta\hat{G}_{\mu\nu})q^b\rangle_S\,. \tag{A.6}$$

Comparing (A.4) and (A.6), (A.5) and (A.3), we see that

$$\begin{aligned}\langle\overline{q}^a\gamma_\sigma\hat{G}_{\mu\nu}D_\eta q^b\rangle_S &= \langle\overline{q}^a\overleftarrow{D_\eta}\gamma_\sigma\hat{G}_{\mu\nu}q^b\rangle_S = -\frac{1}{2}\langle\overline{q}^a\gamma_\sigma(D_\eta\hat{G}_{\mu\nu})q^b\rangle_S\,,\\ \langle\overline{q}^a\gamma_5\gamma_\sigma\hat{G}_{\mu\nu}D_\eta q^b\rangle_S &= -\langle\overline{q}^a\overleftarrow{D_\eta}\gamma_5\gamma_\sigma\hat{G}_{\mu\nu}q^b\rangle_S\,,\\ \langle\overline{q}^a\gamma_5\gamma_\sigma(D_\eta\hat{G}_{\mu\nu})q^b\rangle_S &= 0\,.\end{aligned} \tag{A.7}$$

Now let us write down the general tensor structure of vacuum expectation values (A.1).

$$\begin{aligned}\langle\overline{q}^a_\alpha G^n_{\eta\sigma}D_\tau q^b_\beta\rangle_S &= (t^n)^{ba}\Big(d_1(S_{\tau\eta}\gamma_\sigma - S_{\tau\sigma}\gamma_\eta) + d_2(g_{\tau\eta}S_{\sigma\xi} - g_{\tau\sigma}S_{\eta\xi})\gamma_\xi +\\ &\qquad + id_3\epsilon_{\eta\sigma\xi\chi}S_{\tau\xi}\gamma_\chi\gamma_5 + id_4\epsilon_{\tau\eta\sigma\xi}S_{\xi\chi}\gamma_\chi\gamma_5\Big)_{\beta\alpha}\,,\\ \langle\overline{q}^a_\alpha\overleftarrow{D_\tau}G^n_{\eta\sigma}q^b_\beta\rangle_S &= (t^n)^{ba}\Big(e_1(S_{\tau\eta}\gamma_\sigma - S_{\tau\sigma}\gamma_\eta) + e_2(g_{\tau\eta}S_{\sigma\xi} - g_{\tau\sigma}S_{\eta\xi})\gamma_\xi +\\ &\qquad + ie_3\epsilon_{\eta\sigma\xi\chi}S_{\tau\xi}\gamma_\chi\gamma_5 + ie_4\epsilon_{\tau\eta\sigma\xi}S_{\xi\chi}\gamma_\chi\gamma_5\Big)_{\beta\alpha}\,,\\ \langle\overline{q}^a_\alpha(D_\tau G^n_{\eta\sigma})q^b_\beta\rangle_S &= (t^n)^{ba}\Big(f_1(S_{\tau\eta}\gamma_\sigma - S_{\tau\sigma}\gamma_\eta) + f_2(g_{\tau\eta}S_{\sigma\xi} - g_{\tau\sigma}S_{\eta\xi})\gamma_\xi +\\ &\qquad + if_3\epsilon_{\eta\sigma\xi\chi}S_{\tau\xi}\gamma_\chi\gamma_5 + if_4\epsilon_{\tau\eta\sigma\xi}S_{\xi\chi}\gamma_\chi\gamma_5\Big)_{\beta\alpha}\,,\end{aligned} \tag{A.8}$$

where $d_1,\ldots,d_4$, $e_1,\ldots,e_4$, $f_1,\ldots,f_4$ are unknown constants. All other possible structures can be expressed in terms of these one. Multiplying these equations by $(\gamma_\sigma)_{\alpha\beta}$, $g_{\tau\eta}(\gamma_\rho)_{\alpha\beta}$, $\epsilon_{\eta\sigma\mu\nu}(\gamma_5\gamma_\rho)_{\alpha\beta}$ and so on and using (A.7), we find after simple algebra:

$$\begin{aligned} d_1 &= e_1 = -\frac{1}{2}f_1\,, \qquad & d_2 &= e_2 = -\frac{1}{2}f_2\,, \qquad & f_3 = f_4 = f_5 = 0\,,\\ d_3 &= -e_3\,, & d_4 &= -e_4\,. \end{aligned}$$

One can find also the relation between d_1 and d_2. Multiplying the third of equations (A.8) by $(\gamma_\chi)_{\alpha\beta}g_{\tau\eta}(t^n)^{ab}$, we find:

$$\langle\overline{q}^a_\alpha(D_\tau G^n_{\eta\sigma})q^b_\beta\rangle_S(\gamma_\chi)_{\alpha\beta}g_{\tau\eta}(t^n)^{ab} = -g\langle(\overline{q}\gamma_\chi t^n q)(\overline{q}\gamma_\sigma t^n q)\rangle_S\,.$$

Using factorization hypothesis, $\langle(\overline{q}\gamma_\chi t^n q)(\overline{q}\gamma_\sigma t^n q)\rangle_S$ can be expressed in terms of $\langle\overline{q}^a_\alpha q^b_\beta\rangle_S$, which equals to zero, since the proportional to the external field $S_{\mu\nu}$ tensor structure does not exist. We obtain:

$$d_1 = 3d_2\,.$$

Thus, all vacuum expectation values (A.8) are expressed in terms of three unknown constants (for example, d_1, d_3, d_4).

Therefore, we obtain for the vacuum expectation values (A.8):

$$\begin{aligned}
\langle \overline{q}^a_\alpha G^n_{\eta\sigma} D_\tau q^b_\beta \rangle_S &= (t^n)^{ba} \Bigg(d_1 \left(S_{\tau\eta}\gamma_\sigma - S_{\tau\sigma}\gamma_\eta + \frac{1}{3}(g_{\tau\eta}S_{\sigma\xi} - g_{\tau\sigma}S_{\eta\xi})\gamma_\xi \right) + \\
&\qquad + i d_3 \epsilon_{\eta\sigma\xi\chi} S_{\tau\xi}\gamma_\chi\gamma_5 + i d_4 \epsilon_{\tau\eta\sigma\xi} S_{\xi\chi}\gamma_\chi\gamma_5 \Bigg)_{\beta\alpha}, \\
\langle \overline{q}^a_\alpha \overleftarrow{D}_\tau G^n_{\eta\sigma} q^b_\beta \rangle_S &= (t^n)^{ba} \Bigg(d_1 \left(S_{\tau\eta}\gamma_\sigma - S_{\tau\sigma}\gamma_\eta + \frac{1}{3}(g_{\tau\eta}S_{\sigma\xi} - g_{\tau\sigma}S_{\eta\xi})\gamma_\xi \right) - \\
&\qquad - i d_3 \epsilon_{\eta\sigma\xi\chi} S_{\tau\xi}\gamma_\chi\gamma_5 - i d_4 \epsilon_{\tau\eta\sigma\xi} S_{\xi\chi}\gamma_\chi\gamma_5 \Bigg)_{\beta\alpha}, \\
\langle \overline{q}^a_\alpha (D_\tau G^n_{\eta\sigma}) q^b_\beta \rangle_S &= -2 d_1 (t^n)^{ba} \left(S_{\tau\eta}\gamma_\sigma - S_{\tau\sigma}\gamma_\eta + \frac{1}{3}(g_{\tau\eta}S_{\sigma\xi} - g_{\tau\sigma}S_{\eta\xi})\gamma_\xi \right)_{\beta\alpha}.
\end{aligned} \tag{A.9}$$

For compliteness we express d_1, d_3, d_4 in terms of the unknown vacuum expectation values:

$$\begin{aligned}
d_1 S_{\tau\mu} &= \frac{3}{2^7} \langle \overline{q}\gamma_\sigma \hat{G}_{\mu\sigma} D_\tau q\rangle_S , \\
d_3 S_{\tau\mu} &= \frac{i}{2^8} \epsilon_{\eta\sigma\mu\rho} \Big(3\langle \overline{q}\gamma_5\gamma_\rho \hat{G}_{\eta\sigma} D_\tau q\rangle_S - \langle \overline{q}\gamma_5\gamma_\tau \hat{G}_{\eta\sigma} D_\rho q\rangle_S \Big), \\
d_4 S_{\tau\mu} &= \frac{i}{2^8} \epsilon_{\eta\sigma\mu\rho} \Big(\langle \overline{q}\gamma_5\gamma_\rho \hat{G}_{\eta\sigma} D_\tau q\rangle_S - 3\langle \overline{q}\gamma_5\gamma_\tau \hat{G}_{\eta\sigma} D_\rho q\rangle_S \Big).
\end{aligned}$$

Multiplying now the first of equations (A.9) by $(\gamma_\tau)_{\gamma\beta}(t^n)^{ab}$ and using equation of motion

$$\hat{D}q - S_{\mu\nu} D_\mu \gamma_\nu q = 0 , \tag{A.10}$$

we obtain additional relation:

$$\frac{4}{3} d_1 - d_3 + d_4 = -\frac{g\langle \overline{q}q\rangle^2}{3^3 2^4} . \tag{A.11}$$

Now we consider the vacuum expectation value (A.2)

$$\langle \overline{q}^a_\alpha (D_{\{\sigma} D_\tau D_{\eta\}} q^b_\beta \rangle_S = (c_1 S_{\{\sigma\tau}\gamma_{\eta\}} + c_2 g_{\{\sigma\tau} S_{\eta\}\chi}\gamma_\chi)_{\beta\alpha} \delta^{ab} .$$

Multiplying this relation by $(\gamma_\eta)_{\gamma\beta}$ and taking into account (A.9), (A.10) and the identity $[D_\mu D_\nu] = -igt^n G^n_{\mu\nu}$, we find

$$\begin{aligned}
c_1 &= -3k(1+\epsilon_1) , \\
c_2 &= 3k(1+\epsilon_2) ,
\end{aligned} \tag{A.12}$$

where

$$k = -\frac{ig^2 \langle \overline{q}q\rangle^2}{3^5 2^4} ,$$

$$\epsilon_1 = \frac{ig}{3k}\left(-d_1 + \frac{d_3}{3}\right),$$
$$\epsilon_2 = \frac{ig}{3k}\left(\frac{7}{9}d_1 + d_3\right). \tag{A.13}$$

In (A.12) the first term appears due to $\langle \overline{q}^a_\alpha D_\mu D_\nu \hat{D} q^b_\beta \rangle$, which can be calculated by use of the equation of motion (the factorizable vacuum expectation value), while ϵ_1, ϵ_2, expressed in terms of d_1, d_3, d_4, correspond to contributions of the nonfactorizable vacuum expectation values (i.e. to contributions of the vacuum expectation values, which can not be calculated in such a way).

Thus, vacuum expectation values (A.1), (A.2) are expressed in terms of three unknown parameters, related by (A.11).

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